

UNIT-I

- 1 a) Derive the differential equation of equilibrium in terms of displacement components for plane stress problem in the presence of body forces. [5M]
- b) Explain plane stress and plane strain with examples. [5M]
- 2 a) State Hooke's law and explain about pure shear. [5M]
- b) Explain about components of strain at a point [5M]
- 3 The stress tensor at a point in the body is given by
poisson's ratio is 0.3 for the material, find the strain tensor at this point [10M]
- $$\begin{bmatrix} +600 & -200 & +300 \\ -200 & +200 & +450 \\ +300 & -450 & -400 \end{bmatrix} \times 10^{-6}. \quad E = 2 \times 10^5 \text{ N/mm}^2$$
- 4 The state of stress is at point is given by
 $\sigma_{xx} = 10\text{MPa}$, $\sigma_{yy} = -20\text{MPa}$, $\sigma_{zz} = -10\text{MPa}$
 $\tau_{xy} = -20\text{MPa}$, $\tau_{yz} = 10\text{MPa}$, $\tau_{xz} = -30\text{MPa}$ if [10M]
 $E = 250\text{GPa}$ and $G = 80\text{GPa}$. Find out the corresponding strain components from
- 5 List the six components of strain. Derive the strain components between the same for the different planes. [10M]
- 6 (a) What is Airy's stress function? Discuss the application of stress function approach for solving of two dimensional bending problems. [5M]
- (b) Obtain the relationship between three elastic moduli for plan stress problem. [5M]
- 7 (a) Derive the equations of equilibrium in Cartesian form. [5M]
- (b) Derive stress-strain displacement relations for Cartesian coordinate system. [5M]
- 8 a) Show that $\Delta^2(\sigma_y + \sigma_z) = -\frac{1}{1-\nu} \left[\frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} \right]$ using plain strain condition [5M]
- b) Obtain the compatibility equation for plane stress problem in Cartesian form. [5M]
- 9 a) Develop stress- strain relations for plane stress problems [5M]
- b) Derive the differential equations of equilibrium and compatibility equations in 2-dimensional Cartesian coordinate system. [5M]
- 10 a) For the following strain distribution, verify whether the compatibility condition is satisfied: [5M]
- (i) $\epsilon_{xx} = 3x^2y$, $\epsilon_{yy} = 4y^2x + 10^{-2}$, $\gamma_{xy} = 2xy + 2x^3$
- (ii) $\epsilon_{xx} = py$, $\epsilon_{yy} = px$, $\epsilon_{zz} = 2p(x + y)$
 $\gamma_{xy} = p(x + y)$, $\epsilon_{yz} = 2pz$, $\epsilon_{zx} = 2pz$, where p is a constant.
- b) Explain stress functions with examples [5M]

UNIT-II

- 11 a) Prove that following are Airy's stress function and examine the stress distribution by them: [5M]
- (i) $\phi = Ax^3 - y$.
- (ii) $\phi = Ax^2 - By^2$.
- (iii) $\phi = Ax^2 + Bxy + Cy^2$.
- b) Explain Saint-Venant's principle with example. [5M]
- 12 Determine the stress components and sketch their variation in a region included $y=0, y=d$ and $x=0$ on the side x positive. For the given stress function: [10M]
- $$\phi = \frac{-F}{d^3} xy^2(3d - 2y)$$
- 13 a) Check whether the following
Where $C = \text{constant}$. [5M]
- $$\phi = \frac{q}{8c^3} [x^2(y^3 - 3c^2y + 2c^3)] - \frac{1}{5} y^3(y^2 - 2c^2)$$
- b) What is plane strain? Explain it [5M]
- 14 Investigate what problem is solved by the stress function. [10M]
- $$\phi = \frac{3F}{4C} \left[xy - \frac{xy^3}{3C^2} \right] + \frac{p}{2} y^2.$$

- 15 Show that $\phi = \frac{P}{2\pi} \left[X^2 + Y^2 \operatorname{Arctan} \left(\frac{Y}{X} \right) - XY \right]$ is a stress function and hence determine the expressions for σ_x , σ_y and τ_{xy} [10M]
- 16 A cantilever of length 'L' and depth 2C is of unit thickness. A force of P is applied at the free end. The upper and the lower edges are free from load. Obtain the equation of deflection curve of the beam in the form
Where X is the distance from free end. [10M]
- $$(V)_{x=0} = \frac{PX^3}{6EI} - \frac{PL^2X}{2EI} + \frac{PL^3}{3EI}$$
- 17 Assume the fifth order polynomial degree for the rectangular beam strip and find the Airy's stress function with the different stress components. Analyze the behavior of the beam and draw the stress distribution diagram [10M]
- 18 a) Derive the compatibility conditions for the two dimensional Cartesian coordinates. [5M]
- b) Prove that $\sigma_z = \nu(\sigma_x + \sigma_y)$. Give the practical examples and draw the neat diagram. [5M]
- 19 a) Show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) (\sigma_x + \sigma_y) = 0$ For a 2-D elastic body. [5M]
- b) what is stress function(ϕ)? Show that $\nabla^4 \phi = 0$. [5M]
- 20 a) Discuss the various stress cases obtained by taking third order polynomial as Airy's stress function [5M]
- b) Derive stress-strain displacement relations for Cartesian coordinate system. [5M]

UNIT-III

- 1 Derive the differential equilibrium equation in polar coordinates for two dimensional elastic bodies [10M]
- 2 Starting from fundamentals, derive the expression for hoop and radial stresses for a rotating hollow disc [10M]
- 3 Derive the stress components of a plate with circular hole subjected to uniaxial load [10M]
- 4 Explain generalized solution of the two-dimensional problem in polar coordinates [10M]
- 5 Starting from a suitable stress function for an axially symmetric problem, derive Lamé's expression for radial and hoop stresses in a thick cylinder subjected to internal fluid [10M]
- 6 Derive the equilibrium along with the boundary conditions and compatibility conditions for the two dimensional polar coordinates [10M]
- 7 Determine stress components for the stress function [10M]
- 8 Derive the equilibrium $\phi = A \log r + B r^2 \log r + C r^2 + D$ a compatibility conditions for the two dimensional polar coordinates. [10M]
- 9 a) Obtain the general expression for stresses for an axisymmetric problem. [5M]
- b) Obtain the compatibility expression for two dimensional problem in polar coordinates [5M]
- 10 A curved bar with a constant narrow rectangular cross section and a circular axis is bent in the plane of curvature by couples M applied at the end taking the solution in the [10M]

UNIT-IV

- 11 Derive the equation of equilibrium for 3-D stress state. [10M]
- 12 Determine the principal stress tensor at a point in a material if the strain tensor at a point is given below. And Poisson's ratio 0.3. Define stress invariants also [10M]
- 13 What are the stress invariants? Derive expression for the stress invariants. [10M]
- 14 The state of stress in given at a point by following matrix. Determine principle stresses and principle direction: $\begin{bmatrix} 9 & 6 & 3 \\ 6 & 5 & 2 \\ 3 & 2 & 4 \end{bmatrix}$ [10M]
- 15 A point P in a body is given by $\begin{bmatrix} 9 & 6 & 3 \\ 6 & 5 & 2 \\ 3 & 2 & 4 \end{bmatrix}$. Determine the total stress, normal stress and shear stress on a plane which is equally inclined to all the three axes [10M]
- 16 a) Derive the eq $Z = \begin{bmatrix} 100 & 100 & 100 \\ 100 & -50 & 100 \\ 100 & 100 & -50 \end{bmatrix} \text{mN/mm}^2$ displacements. [5M]
- b) Explain the ϵ [5M]
- 17 The state of stress at a point is given by following stress tensor. Calculate the stress invariants, magnitude and direction of principal stresses [10M]
- 18 The stress tensor at a particular point is given by: Calculate for the plane having direction cosines, $a_{nx} = \frac{1}{\sqrt{2}}$, $a_{ny} = \frac{1}{\sqrt{2}}$ & $a_{nz} = \frac{1}{\sqrt{2}}$ [10M]
- 19 a) Derive the expression for principal stresses in three dimensions. [5M]
- b) What is meant by Homogenous deformation? Explain with examples [5M]
- 20 Derive the compatibility relation of strain in a 3D elastic body. What is its significance? [10M]

UNIT-V

- 21 Derive an expression for torsion of a bar of narrow rectangular cross section [10M]
Derive the governing equation and the boundary for non-circular section subjected to
- 22 torque load [10M]
- 23 Explain and derive the equation for the Prandtl's membrane analogy [10M]
- 24 Explain the membrane analogy, applied to a narrow rectangular section. [10M]
Obtain the expression for the maximum shear stress of a shaft of elliptical cross section
- 25 having major [10M]
A rectangular beam of width $2a$ and $2b$ is subjected to torsion. Derive the equation
- 26 for obtaining maximum shear stress. [10M]
A steel I-section of flange $200\text{mm} \times 12\text{mm}$ and web $570\text{mm} \times 8\text{mm}$ is subjected to a
- 27 pure torque. If the maximum shear stress in the material is 100 N/mm^2 , Find the torque [10M]
- 28 Derive the differential equation $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 2G\theta$, for torsion problem in elasticity, [10M]
Show that in torsion $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 2G\theta$ is satisfied along the boundary of the
- 29 Rigidity, J-polar moment of inertia and θ - angular twist for unit length [10M]
- 30 write short notes on.
- a) Distortion Energy. [3M]
- b) Prandtl membrane analogy [4M]
- c) Saint venant's principle [3M]

Subject Name with Code: Theory of Elasticity (16CE2002)

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